# Low Complexity MMSE MIMO Detector for 3GPP LTE Downlink

Shobana S, Joseph Gladwin S

**Abstract**— In this paper a low complexity Minimum-Mean Squared Error (MMSE) detector for Multiple-Input-Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system is addressed. MIMO-OFDM is a promising technique for modern wireless communication that demand high data rate for applications such as Third Generation Partnership Project Long Term Evolution (3GPP LTE). OFDM divides a wideband frequency selective wireless fading channel into narrowband flat fading sub-channels and is the most popular multicarrier scheme because it can be implemented by efficient Fast Fourier Transform (FFT)/Inverse Fast Fourier Transform (IFFT) techniques. Basically in OFDM systems, interpolation concepts are included to reconstruct the channel response so that the performance of channel estimation gets improved. The proposed detector uses division-free matrix inversion algorithm for the computation of MMSE matrix on pilot subcarriers which reduces the overall complexity by combining computations of channel estimation and MIMO detection. Modifying the banachiewicz matrix inversion formulae in the algorithm greatly results in the reduction of computational complexity of the system. The LTE system is built using MathWorks MATLAB and the Bit Error Rate (BER) performances are analyzed.

**Index Terms**— Multiple-Input-Multiple-Output (MIMO), Orthogonal Frequency Division Multiplexing (OFDM), Minimum-Mean Squared Error (MMSE), Third Generation Partnership Project Long Term Evolution (3GPP LTE), interpolation ,wireless communication,channel estimation.

◆

## 1 INTRODUCTION

OVER the past few years, there has been increasing emphasis on extending the services available on wired public telecommunications networks to mobile/movable nonwired telecommunications users. Wireless communication systems are becoming increasingly attractive because of the flexible connectivity and ability of high data rate transmission. Therefore, mobile communication systems are one of the most promising systems utilizing the multipath effects to support efficient transmission and provide high throughput. OFDM is an efficient way to deal with multipath for a given delay spread and makes single-frequency networks possible, which is especially attractive for broadcasting applications. 3GPP LTE [13] is a modern wireless communication standard that incorporates MIMO technology in order to yield significantly high data rate for mobile phones and data terminals. MIMO communication uses multiple antennas at both sides of a wireless link which increases the channel capacity. OFDM is the key technology that enables MIMO technique. In OFDM, the digital signals are encoded by using multiple subcarriers where the symbols are get distributed over these carriers. These transmitted symbols are get detected by the MIMO receiver which needs high computational power to recover the originally transmitted symbols.

Optimal MIMO detectors such as Maximum Likelihood (ML) or Maximum a Posteriori Probability (MAP) detector's computational complexity increases when the number of antennas employed and modulation order goes higher.Zero forcing (ZF) and minimum-mean-squared error (MMSE) are the linear detectors which makes an effort to cancel channel effects on transmitted symbols by applying the inverse effects. Normally, in OFDM-based transmission the referred MIMO detection schemes performs MIMO processing on each sub-carrier which demands high computational complexity and high-power consumption in order to meet the data rate of modern wireless standards. Few works there are available in the literature which tried to reduce the computational complexity of OFDM MIMO detectors by interpolation.

In this paper, a modification in conventional banachiewicz matrix inversion formulae is proposed for the design of MMSE MIMO detector. The proposed detector reduces the overall computational complexity by combining computations of OFDM channel estimation and MIMO detection. The proposed MIMO detection algorithm computes MMSE matrix only on pilot and pseudo pilot sub-carriers and obtains the MMSE matrix of data sub-carriers by interpolation.

The algorithm is developed by introducing a modification in conventional banachiewicz matrix inversion formula [12] which does not require costly division operation at the matrix inversion stage. OFDM channel estimation is getting combined with MIMO detection by using the division-free property of the proposed modified banachiewicz matrix inversion formula and polynomial property of OFDM MIMO channel. This combination reduces computational complexity of OFDM MMSE MIMO detection when compared with conventional tone by tone detection.

---

- *Shobana S is currently pursuing masters degree program in VLSI Design at SSN College of Engineering, Anna University, Kalavakkam, Chennai,. E-mail: shobananth293@gmail.com*
- *Joseph Gladwin S is currently working as Assistant Professor in Electronics and Communication Department at SSN College of Engineering, Anna University, Chennai, Kalavakkam.*

ISSN 2229-5518

## 2 SYSTEM MODEL AND REQUIREMENTS

### 2.1 LTE Overview

LTE is a modern wireless communication standard for broadband system for mobile which improves the performance in terms of coverage, speed, spectral efficiency and throughput [9]. LTE design is framed in order to meet the carrier needs for high-speed data and multimedia broadcast services. LTE is also expected to improve spectral efficiency in 3Gnetworks.
Multiple antennas for reception and transmission at the base station and in User End (UE) are a key enabler of the high performance offered by 3rd Generation Partnership Project (3GPP) LTE. The LTE physical layer (PHY) is a highly efficient means of conveying both data and control information between an enhanced base station (eNodeB) and mobile user equipment (UE). The LTE PHY employs some advanced technologies that are new to cellular applications. These include OFDM and MIMO data transmission. The basic LTE model is shown in Fig.1

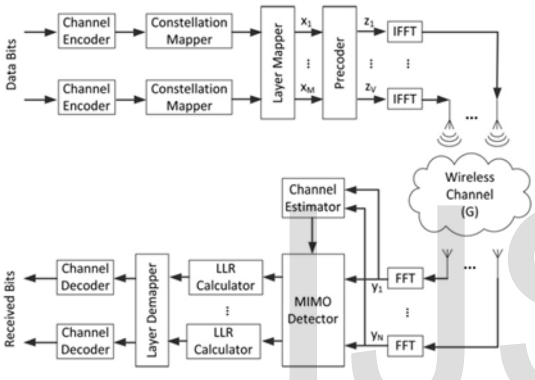

Fig. 1.LTE system Model [3]

OFDM is an excellent choice of multiplexing scheme for the 3GPP LTE downlink. OFDM involves added complexity in terms of resource scheduling; it is extensive to packet-oriented approaches in terms of efficiency and latency [11]. Normally in OFDM, a certain number of subcarriers are allocated to the users for a fixed time. These are mentioned as physical resource blocks (PRBs) in the LTE specifications. So, PRBs have both a time and frequency dimension. The scheduling function at the 3GPP base station deals the allocation of PRBs.

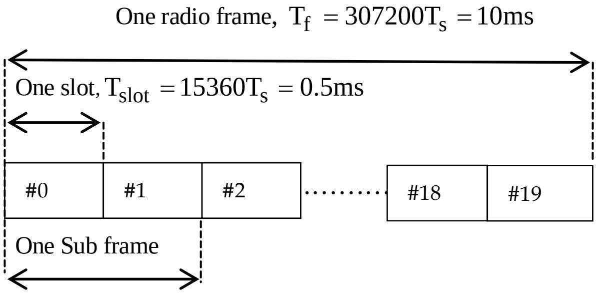

Fig. 2.LTE Generic Frame Structure

From Figure 2, the LTE frames are 10 msec in duration, each frame is divided into 10 sub frames, and each sub frame being 1.0 msec long, finally each sub frame is further divided into two slots of each one is 0.5 msec duration. Each slot consists of either 6 or 7 ODFM symbols, depending on whether the normal or extended cyclic prefix is used [10]. The total number of available subcarriers relies on the overall transmission bandwidth of the system. From [10], for one slot (0.5 of msec) in duration, a PRB is consists of 12 consecutive subcarriers. A PRB is the smallest element of resource allocation assigned by the base station scheduler.

The transmitted downlink signal consists of NBW subcarriers for duration of Nsymb OFDM symbols. It can be represented by a resource grid. Each box in the grid constitutes a single subcarrier for one symbol period and is referred to as a resource element. Note that in MIMO applications, a resource grid is available for each transmitting antenna. LTE does not employ a PHY preamble to facilitate carrier offset estimate, channel estimation, timing synchronization etc. Instead, special reference signals are embedded in the PRBs as shown in Figure 3. Reference signals are transmitted during the first and fifth OFDM symbols of each slot when the short CP is used and during the first and fourth OFDM symbols when the long CP is used.

Note that reference symbols are transmitted every sixth subcarrier. Further, reference symbols are staggered in both time and frequency. The channel response on subcarriers bearing the reference symbols can be computed directly. Interpolation is used to estimate the channel response on the remaining subcarriers.

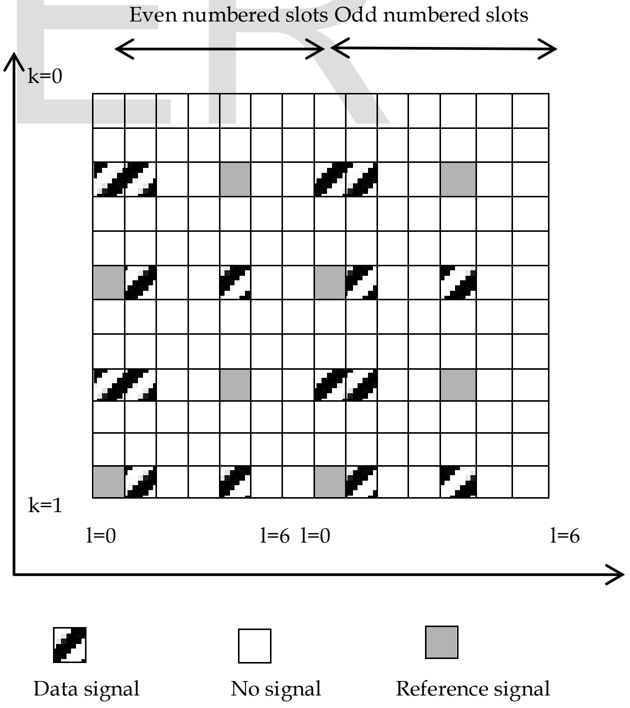

Fig. 3.LTE Reference Signals are interspersed among Resource Elements [3]

### 2.2 System Specifications

Consider a OFDM MIMO system as specified in 3GPP LTE standard [13,14] with M number of transmit antennas and N number of receiving antennas such that $(M, N) = 4$. The MIMO system is modeled as $y = Hx + n$, where y and x is the receiver and transmit vectors, H and n are the channel matrix and noise vector respectively. The transmitted vector is given as $x = [x_1, x_2, x_3 .... x_M]^T$ and then the x vector is then pre-coded as $z = [z_1, z_2, z_3 .... z_V]^T$ given in (1) to obtain the transmit vector

$$Z = VX \tag{1}$$

V is $V \times M$ pre-coder matrix chosen from LTE codebook. The transmitted symbols are affected by MIMO channel and Additive White Gaussian Noise (AWGN). The input–output relation of the MIMO system over the $p^{th}$ OFDM sub-carrier is as follows

$$y_p = G_p Z_p + n_p \tag{2}$$

Where $y = [y_1, y_2 ..... y_N]^T$ such that $(N \geq V)$ is the received vector, G is the physical channel matrix of order $N \times V$, $n = [n_1, n_2 ..... n_N]^T$ is independent and identically distributed zero-mean complex Gaussian noise with variance per entry, and P is the total number of sub-carriers in one OFDM symbol. The input–output relation of the MIMO system can be rewritten as

$$y_p = H_p Z_p + n_p \text{ , } p = 1, 2 ... P \tag{3}$$

Where H is the N ×M matrix which is defined by,

$$H = GV \tag{4}$$

## 3 INTERPOLATION-BASED MMSE MIMO DETECTION

The matrix used in the MMSE detection can be described in the following and for simplicity the subscript p from $H_p$ has been neglected.

$$W = \left(H^H H + \sigma^2 I_M\right)^{-1} H^H \tag{5}$$

$\sigma^2$ Is the variance, $H^H$ is the Hermitian Transpose of the matrix H. The symbols are computed by using this matrix,

$$\hat{x}_i = \frac{1}{u_i} w_i y = x_i + \frac{1}{u_i} w_i n, i = 1, 2 ... M \tag{6}$$

$$u_{(i)} = w_i h_i \tag{7}$$

$w_i$ and $h_i$ in (7) denotes the $i^{th}$ row of W and $i^{th}$ column of H. The log-likelihood ratio (LLR) for the $b^{th}$ bit in the $i^{th}$ detected symbol, $L_{b,i}$ is approximated by the following equation

$$L_{b,i} = \rho_i \lambda_{b,i} \text{ , } b = 1, 2 ... Q \tag{8}$$

where Q is number of bits per symbol. $\lambda_{b,i}$ was computed as in

(9) in [3].

$$\rho_{i=\frac{u_i}{1-u_i}} \tag{9}$$

The response of the MIMO OFDM channel is given as

$$\sum_{l=0}^{L-1} H_\ell e^{-2\pi\left(\frac{p}{P}\right)\ell} \text{ , } p = 1, 2 .... P \tag{10}$$

The above equation is a polynomial matrix of degree L-1.

The inverse of a polynomial matrix is not always polynomial and it cannot be interpolated [6], and from (10) the multiplication and addition operations on channel matrix does not remove polynomial property. By this fact and considering the properties of MMSE MIMO detection algorithm, we can modify the banachiewicz formula for matrix inversion to avoid division operation.

Consider a 4x4 matrix R given as,

$$R = \left(H^H H + \sigma^2 I_M\right) \tag{11}$$

R can be partitioned as follows,

$$R = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{12}$$

Where A, B and C are the 2x2 sub matrices from R.

The inverse of the partitioned matrix is given by Banachiewicz matrix inversion formula [12] such that R is a Hermitian positive definite matrix.

$$R^{-1} = \begin{bmatrix} T^{-1} & -T^{1} C B^{-1} \\ \left(-T^{-1} C B^{-1}\right)^H & B^{-1} + B^{-1} C^H T^{-1} C B^{-1} \end{bmatrix} \tag{13}$$

Where ‘T’ is the Schur Compliment and it is given by

$$T = A - CBC^H \tag{14}$$

$T^{-1}$ and $B^{-1}$ are given by the general matrix inversion formula.

Considering the following change of variable,

$$T \rightarrow \det(B) T \tag{15}$$

On applying the above change of variable, the matrix T can be rewritten as follows

$$T = \det(B) A - C \tilde{B} C^H \tag{16}$$

where $\tilde{B}$ is the cofactor of the matrix B and (12) can be given by general inversion formula as follows

$$R^{-1} = \frac{1}{\Delta} \hat{R} \tag{17}$$

where

$$\hat{R} = \begin{bmatrix} \det(B)^2 \tilde{T} & -\det(B)\tilde{T}C\tilde{B} \\ \left(-\det(B)\tilde{T}C\tilde{B}\right)^H & \det(T)\tilde{B} + \tilde{B}C^H\tilde{T}C\tilde{B} \end{bmatrix} \qquad (18)$$

$$\Delta = \det(B) \times \det(T) \qquad (19)$$

From the above equations it is noted that inversion of the matrix R is required in (5) and $\Delta$ has no contribution in detection of symbols in (6), so that it gets neglected from both numerator and denominator in the calculation of (6). Therefore in 17) division by $\Delta$ is not needed.

For the computation of $\hat{R}$ no division operation is required, the matrix $\hat{R}$ maintains its polynomial of degree 14L-1 and can be interpolated. The matrix $\hat{R}$ can be given as follows

$$\hat{R}\left( e^{2\pi\left(\frac{p}{P}\right)} \right) = \sum_{l=0}^{14L-1} \tilde{R}_\ell e^{-2\pi\left(\frac{p}{P}\right)\ell} \qquad p = 1, 2\ldots P \qquad (20)$$

In every Resource Block (RB), time domain interpolation is performed to create four pilot carriers in each OFDM symbol with that RB [1] and thereafter interpolation based MMSE MIMO detection is carried out in frequency domain [3]. The interpolation based MMSE MIMO detection algorithm for 4x4 MIMO systems is explained as follows.

Initially, a channel matrix $H_p$ of four pilot sub-carriers for each OFDM symbol is required such that $p = 1$ to 4. Using (11) the partitioned matrix $R_p$ was computed. The matrix $W_p$ which is used in the MIMO detection was computed by using (18). Values of $u_{p,i}$ were computed from (7). Interpolate each two consecutive $W_p, \Delta_p, u_{p,i}$ to find $W_d, u_{d,i}, \Delta_{d,i}$ of the eight remaining sub-carriers in each OFDM symbol within an RB such that'd' varies from 1 to12 which is the total number of symbols and 'i' varies from 1 to4 The detected symbols were computed from (6). Finally, LLR has been computed using the (8).

From the algorithm, it is noted that the MMSE matrix on pilot sub-carriers is computed first and then the interpolation of the computed MMSE matrix has been done next to obtain the MMSE matrix of data sub-carriers. This is possible because of the polynomial property of computed $\hat{R}$ matrix [3]. Instead of MMSE processing on 12 sub-carriers it is performed only on 4 which reduce the overall computational complexity of the system.

## 4 SIMULATIONS AND DISCUSSIONS

BER simulation performance is done by using the system parameters which are set based on the LTE standard as given in [13] are summarized in Table 1. MathWorks MATLAB has been used to create the LTE system and the BER performances were analyzed.

TABLE 1
System and Channel Parameters for Simulation

| *Parameters* | *Specifications* |
|---|---|
| Bandwidth | 5 MHZ |
| Frame Structure | TD-LTE |
| No. of Transmitting Antenna x Receiving Antenna | 4 x 4 |
| PRB Number | 25 |
| Sub-carrier Spacing | 15KHz |
| Channel Model | EPA 5 |
| No.of Samples | 28800 |
| Channel Estimation | MMSE-Algorithm |
| Input SNR | 5 to 40dB |
| Modulation | 16,64-QAM |
| FFT Size | 512 |

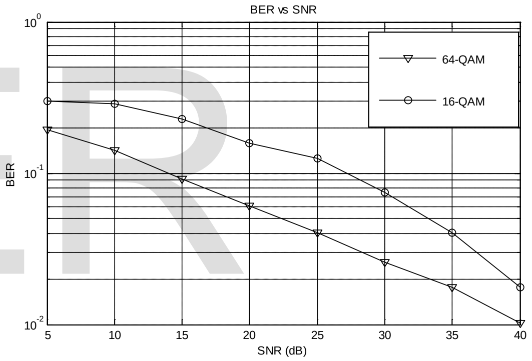

Fig. 4.BER Performance of 16 & 64 QAM

## 5 CONCLUSIONS

In this paper, MMSE MIMO detection algorithm for pilot assisted 4x4 MIMO OFDM systems such as 3GPP LTE using MatLab is presented. Based on the algorithm, initially the MMSE matrix is computed and the resulted MMSE matrices are then interpolated only on pilot carriers to find the MMSE matrix of data carriers. MMSE processing is performed only on 4 out of 12 sub-carriers; so that complexity of the system is reduced. The performance of the system is analyzed over a multipath fading channel in terms of BER. The analysis shows the BER performance of the 16 &64 QAM modulations for 5MHZ bandwidth which is showed in the results.

## References

[1] Chiu, P.-L., Huang, L.-Z., Chai, L.-W., Huang, Y.-H.:'Interpolation-based QR decomposition and channel estimation processor for

MIMO-OFDM system', *IEEE Trans. Circuits Syst. I*, 2011, 58, pp. 1129–1141.

[2] Zhang, J., Huang, X., Suzuki, H., Chen, Z.: 'Gaussian approximation-based interpolation for channel matrix inversion in MIMO-OFDM systems', *IEEE Trans. Wirel. Commun*. 2013, 12, (3), pp. 1407–1417.

[3] Amin Salari, Sied Mehdi Fakhraie and Aliazam Abbasfar, (2014), 'Algorithm and FPGA implementation of interpolation-based soft output MMSE MIMO detector for 3GPP LTE', in IET Communications, vol.8, Issue 4, pp. 492–4s99.

[4] Ma, L., Dickson, K., McAllister, J., McCanny, J.: 'QRdecomposition-based matrix inversion for high performance embedded MIMO receivers', *IEEE Trans. Signal Process*., 2011, 59, (4),pp. 1858–1867

[5] Eilert, J., Di, W., Dake, L.: 'Implementation of a programmable linear MMSE detector for MIMO-OFDM'. IEEE Int. Conf. Acoustics, Speech and Signal Processing (ICASSP), 2008, pp. 5396–5399.

[6] Borgmann, M., Bolcskei, H.: 'Interpolation-based efficient matrix inversion for MIMO-OFDM receivers'. Proc. IEEE Asilomar Conf. on Signals, Systems and Computers, 2004, vol. 2, pp. 1941–1947.

[7] Cescato, D., Borgmann, M., Bolcskei, H., Hansen, J., Burg, A.: 'Interpolation-based QR decomposition in MIMO-OFDM systems'. Proc. IEEE Int. Workshop on Signal Proc. Adv. Wireless Communications, 2005, pp. 945–949

[8] Jin Wang, Muqing Wu and Feng Zheng, (2010), 'The Codebook Design for MIMO Precoding Systems in LTE and LTE-A', Published in Wireless Communications Networking and Mobile Computing (WiCOM), 6th International Conference.

[9] Karl Werner,Johan Furuskog,Mathias Riback,Bo Hagerman,(2010), 'Antenna Configurations for 4x4 MIMO in LTE – Field Measurements', Access Technologies and Signal Processing, Ericsson Research, Ericsson AB, Sweden

[10] Jim Zyren,(2007),'Overview of the 3GPP Long Term Evolution Physical Layer', Technical White Paper edited by Dr. Wes McCoy.

[11] Al-Dulaimi and Anwer,(2013),'Self-Organization and Green Applications in Cognitive Radio Networks' Technology & Engineering

[12] Meyer, C.D.: 'Matrix analysis and applied linear algebra' (Society for Industrial and Applied Mathematics (SIAM), 2000)

[13] Third Generation Partnership Project (3GPP); Technical Specification Group Radio Access Network: 'Evolved Universal Terrestrial Radio Access E-UTRA; Physical Channels and Modulation TS 36.211 (Version 10.5.0)', Tech. Rep., 2012.

[14] Third Generation Partnership Project (3GPP); Technical Specification Group Radio Access Network: 'Evolved Universal Terrestrial RadioAccess E-UTRA; Multiplexing and Channel Coding TS 36.212 (Version10.4.0)', Tech. Rep., 2012.